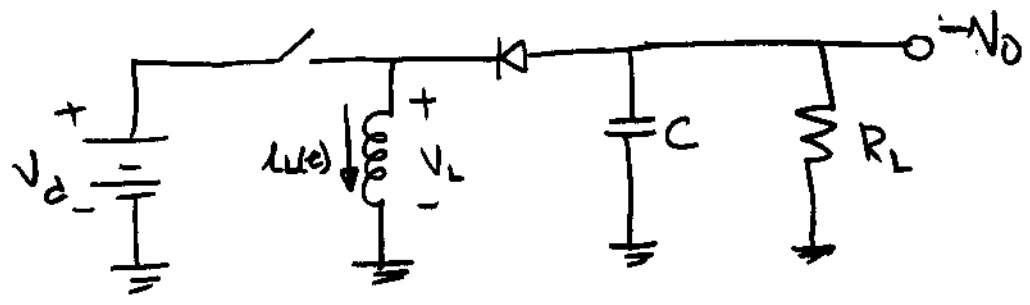
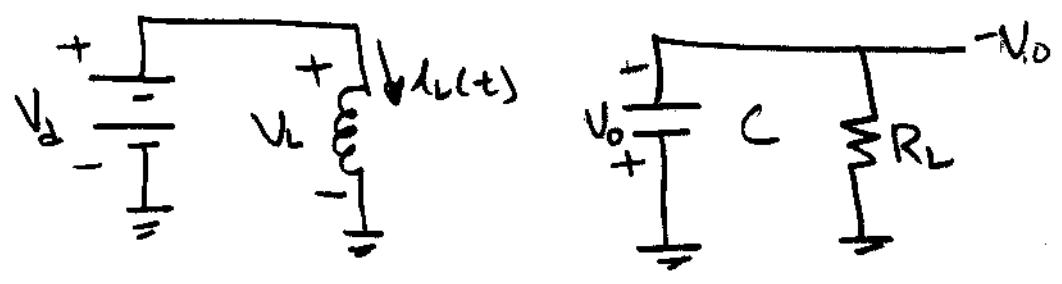


# Polarity Inverting Regulator OR Buck-Boost Regulator



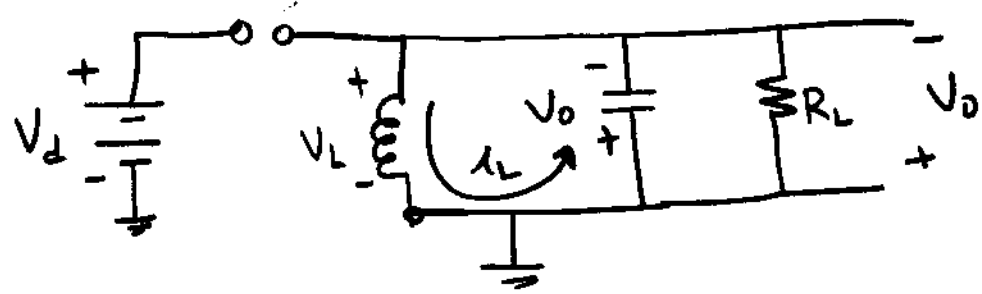
During  $t_{on}$  : Switch Closed  
 $D = \text{off}$



- Capacitor supplies power to load

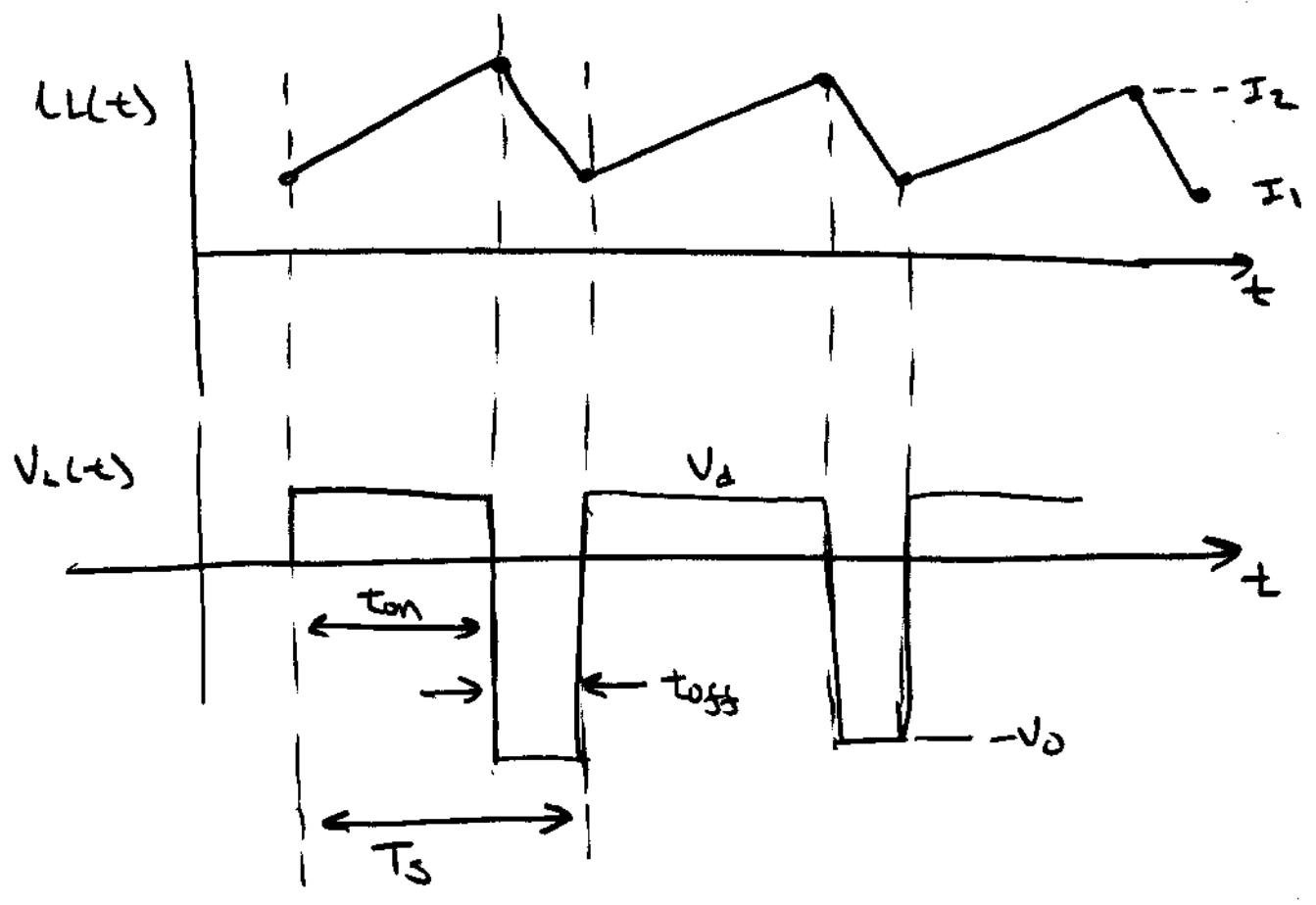
$$V_L = V_d$$

During  $t_{off}$  : Switch = off  
 $D = on$



- $V_L = -V_o$
- Capacitor supplies power to Load
- Inductor dumps energy to capacitor

Continuous mode operation



- In Continuous mode  $I_L > 0$   
and  $t_{on} + t_{off} = T_s$

- Since in steady state, the average current over one cycle is constant,

$$\int_{T_s} V_L(t) dt = 0$$

OR  $V_d t_{on} + (-V_o t_{off}) = 0$

OR  $V_d t_{on} - V_o (T_s - t_{on}) = 0$

OR  $\boxed{\frac{V_o}{V_d} = \frac{t_{on}}{T_s - t_{on}} = \frac{D}{1-D}}$  ;  $D = \frac{t_{on}}{T_s}$  (1)

Assuming a lossless circuit,  $P_d = P_o$ , OR

$$V_d I_d = V_o I_o$$

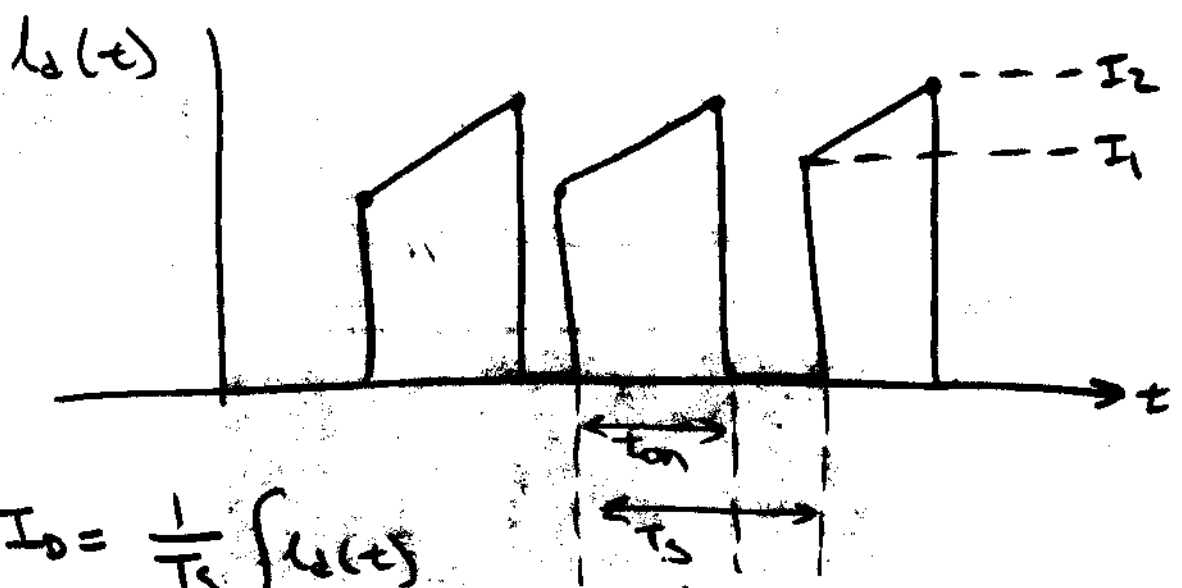
Where  $I_d$  = average input current and  $I_o$  = average output current

Since  $P_o = P_i$

$$\frac{I_o}{I_d} = \frac{V_d}{V_o} = \frac{1-D}{D} \quad (2)$$

- Now find the average input current

-  $V_d$  only supplies current during  $t_{on}$



$$I_o = \frac{1}{T_s} \int i_d(t)$$

$$I_o = \left( \frac{I_1 + I_2}{2} \right) \frac{t_{on}}{T_s} = \left( \frac{I_1 + I_2}{2} \right) D$$

Sub into eq (2)

$$I_0 = \frac{1-D}{D} I_2$$

$$I_0 = \frac{1-D}{D} \left( \frac{I_1 + I_2}{2} \right) D$$

$$\frac{I_1 + I_2}{2} = \frac{2I_0}{1-D}$$

③

Next use  $v_L(t) = L \frac{di}{dt}$

$$i_L(t) = \frac{1}{L} \int v_L dt + i.c.$$

$$I_2 = \frac{1}{L} \int_0^{ton} v_d dt + I_1$$

$$I_2 - I_1 = \frac{V_d ton}{L}$$

④

- Next, Find the boundary between  
CONTINUOUS + DISCONTINUOUS mode.

$\Rightarrow$  solve for  $I_1$

$$I_1 + I_2 = \frac{2I_0}{1-D}$$

$$(-) \quad I_2 - I_1 = \frac{V_d t_{on}}{L}$$

---


$$2I_1 = \left( \frac{2I_0}{1-D} \right) - \frac{V_d t_{on}}{L}$$

For the boundary btw continuous and  
discontinuous mode, solve for  $I_1 = 0$

$$0 = \frac{2I_0}{1-D} - \frac{V_d t_{on}}{L}$$

$$I_0 = \left( \frac{V_d t_{on}}{2L} \right) (1-D)$$

OR

$$\boxed{I_0 \geq \frac{V_d t_{on}}{2L} (1-D)}$$

## Summary

BUCK-BOOST in  
continuous mode

$$V_o = V_d \left[ \frac{D}{1-D} \right]$$

$$I_o = I_o \left[ \frac{1-D}{D} \right]$$

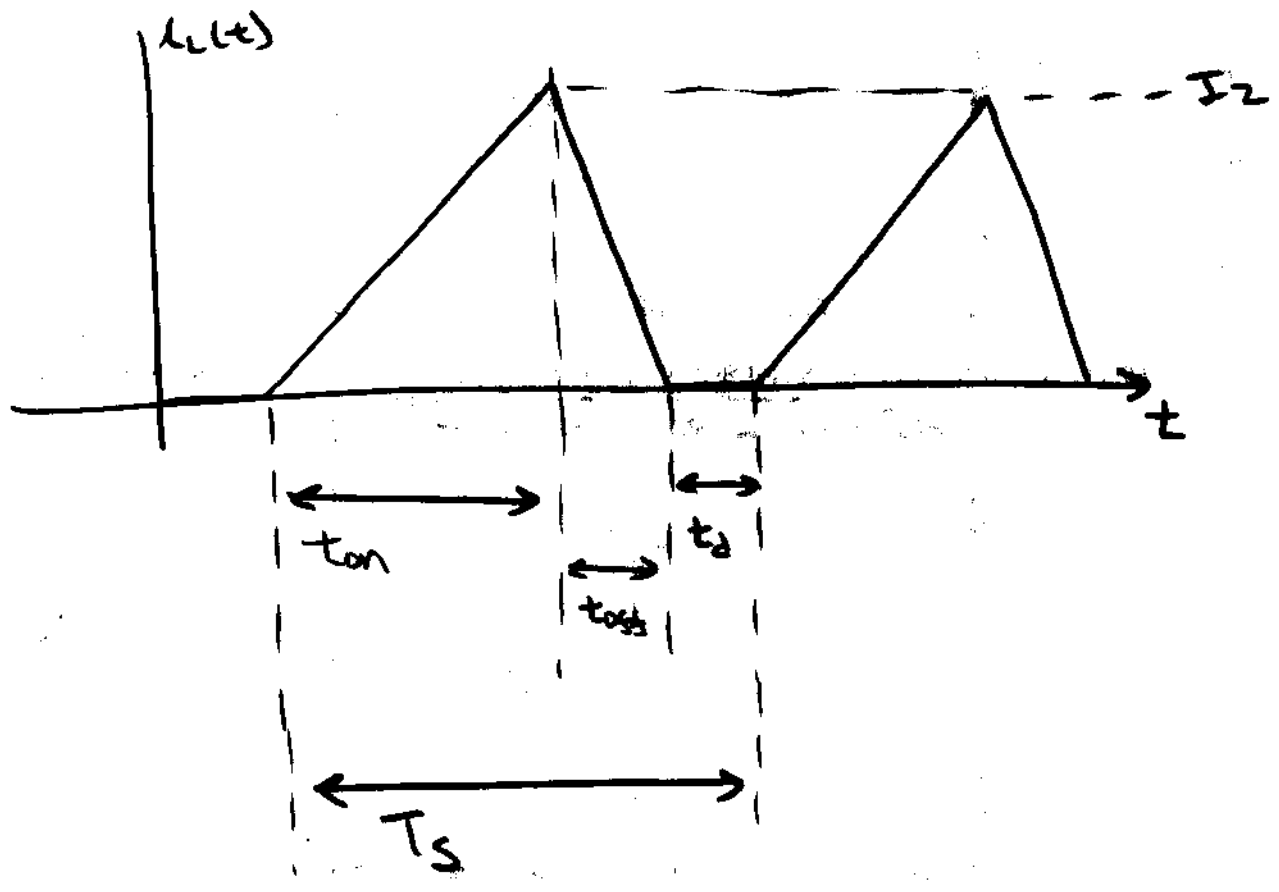
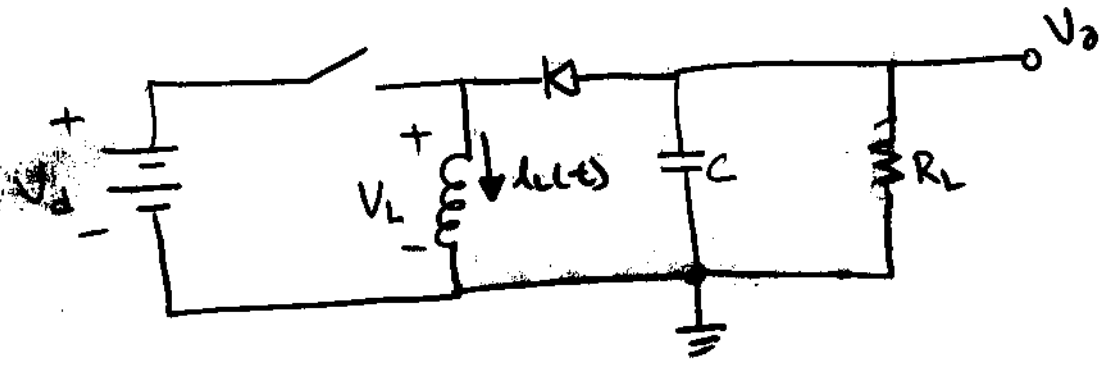
$$\frac{I_1 + I_2}{2} = \frac{I_o}{1-D}$$

$$I_2 - I_1 = \frac{V_d t_{on}}{L}$$

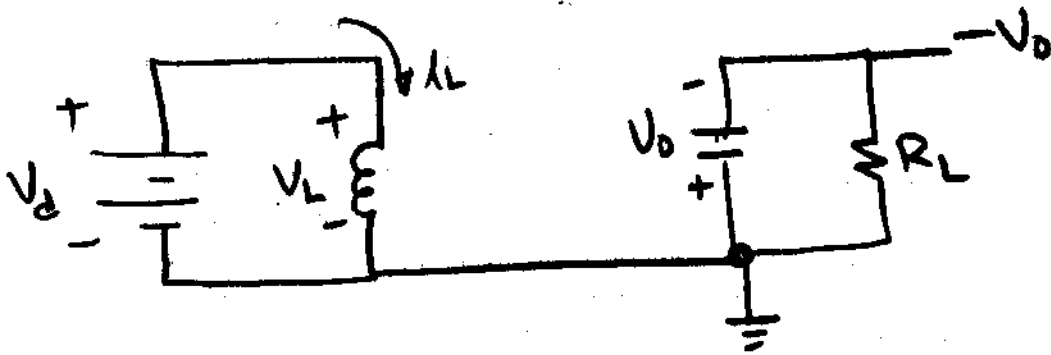
$$I_o \geq \frac{V_d t_{on}}{2L} (1-D)$$

# BUCK-BOOST Regulator

## Discontinuous mode



During  $t_{on}$  we have



For the inductor,  $V_L = L \frac{di_L}{dt}$

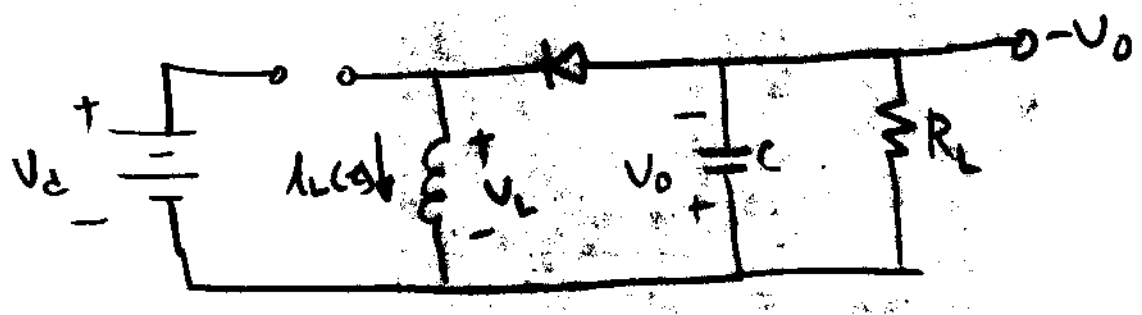
$$i_L(t) = \frac{1}{L} \int V_L(t) dt + i_{L.C.}$$

OR

$$I_2 = \frac{1}{L} \int_0^{t_{on}} V_d dt$$

$$I_2 = \frac{V_d t_{on}}{L} \quad \text{①}$$

During  $t_{off}$  We have



- Diode is Ideal
- D = ON
- $V_L = -V_o$

For the inductor  $V_L = L \frac{di_L}{dt}$

$$i_L(t) = \frac{1}{L} \int V_L(t) dt + I_2$$

OR

$$0 = \frac{1}{L} \int_{t_{on}}^{t_{on} + t_{off}} (-V_o) dt + I_2$$

$$I_2 = \frac{V_o t_{off}}{L} \quad \text{②}$$

- To ensure discontinuous mode operation

Choose  $T_d = 0.2 T_s$

$$\Rightarrow \boxed{t_{on} + t_{off} \leq 0.8 T_s} \quad (3)$$

### Energy balance

- Energy dissipated by load during a cycle

$$E_o = V_o I_o T_s$$

- During  $t_{off}$ , all energy stored in  $L$  is delivered to Load

$$E_L = \frac{1}{2} L I_2^2$$

but from (1),  $I_2 = \frac{V_c t_{on}}{L}$

so

$$E_L = \frac{1}{2} L \left( \frac{V_d^2 t_{on}^2}{L^2} \right)$$

$$E_L = \frac{V_d^2 t_{on}^2}{2L}$$

Energy balance

$$E_L = E_o$$

$$\frac{V_d^2 t_{on}^2}{2L} = V_o I_o T_s$$

$$V_o = \frac{V_d^2 t_{on}^2}{2L I_o T_s} \quad (4)$$

# BUCK-BOOST Summary

## Discontinuous mode

$$\begin{aligned}
 I_2 &= \frac{V_d t_{on}}{L} \\
 I_2 &= \frac{V_o t_{off}}{L}
 \end{aligned}
 \Rightarrow \frac{t_{on}}{t_{off}} = \frac{V_o}{V_d}$$

$$t_{on} + t_{off} \leq 0.8 T_s$$

$$V_o = \frac{V_d^2 t_{on}^2}{2 L I_o T_s}$$

$$I_o \leq \frac{V_d t_{on}}{2L} (1-D) \quad \text{For discontinuous mode}$$

# EE 456

## Buck-Boost Regulator Design Discontinuous Mode Operation

Define useful units for Electrical Engineering

$$\Omega \equiv \text{ohm} \quad k\Omega \equiv 1000 \cdot \text{ohm} \quad M\Omega \equiv 1000000 \cdot \text{ohm}$$

$$F \equiv \text{farad} \quad pF \equiv 10^{-12} \cdot \text{farad} \quad \mu F \equiv 10^{-6} \cdot \text{farad} \quad nF \equiv 10^{-9} \cdot \text{farad} \quad mF := F \cdot 10^{-3}$$

$$ms \equiv .001 \cdot \text{sec} \quad \text{minute} \equiv 60 \cdot \text{sec} \quad \text{hour} \equiv 60 \cdot \text{minute} \quad \text{AmpHour} \equiv 1 \cdot \text{amp} \cdot 1 \cdot \text{hour} \quad \mu s \equiv .001 \cdot ms$$

$$mV \equiv \frac{\text{volt}}{1000} \quad kV \equiv 1000 \cdot \text{volt} \quad mA \equiv \frac{\text{amp}}{1000} \quad \mu A \equiv \frac{\text{amp}}{1000000} \quad mW \equiv \frac{\text{watt}}{1000}$$

$$\mu_o \equiv 1.26 \cdot 10^{-6} \cdot \frac{\text{henry}}{\text{m}} \quad \epsilon_o \equiv 8.854 \cdot 10^{-12} \cdot \frac{\text{farad}}{\text{m}} \quad cc \equiv \text{cm}^3 \quad c \equiv (3 \cdot 10)^8 \cdot \frac{\text{m}}{\text{sec}}$$

$$nH \equiv 1 \cdot 10^{-9} \cdot \text{henry} \quad \mu H \equiv 10^{-6} \cdot \text{henry} \quad kHz \equiv 1000 \cdot \text{Hz} \quad MHz \equiv 1000000 \cdot \text{Hz}$$

$$\text{rad\_per\_sec} \equiv 2 \cdot \pi \cdot \text{Hz}$$

$$\text{cup} \equiv 8 \cdot \text{fl\_oz} \quad \text{tablespoon} \equiv \frac{1}{2} \cdot \text{fl\_oz} \quad \text{Tbsp} \equiv \text{tablespoon} \quad \text{Vt} := .0258 \cdot \text{volt}$$

$$\text{teaspoon} \equiv \frac{1}{3} \cdot \text{Tbsp} \quad \text{tsp} \equiv \text{teaspoon}$$

$$\% := \frac{1}{100}$$

$$\text{Specify Input Voltage } V_D := 5 \cdot \text{volt}$$

$$\text{Specify Output Voltage } V_o := -12 \cdot \text{volt}$$

$$\text{Specify Switching Frequency } F_S := 20 \cdot \text{kHz}$$

$$T_S := \frac{1}{F_S}$$

$$T_S = 50 \cdot \mu s$$

Specify the Max output Current

$$I_o := 1 \cdot \text{amp}$$

Find Ton and Toff

$$T_{\text{off}} := 1 \cdot \mu s$$

$$T_{\text{on}} := 1 \cdot \mu s$$

Given

$$\frac{T_{\text{on}}}{T_{\text{off}}} = \frac{|V_o|}{V_D}$$

$$T_{\text{on}} + T_{\text{off}} = 0.8 \cdot T_S$$

$$\begin{bmatrix} T_{\text{on}} \\ T_{\text{off}} \end{bmatrix} := \text{find}(T_{\text{on}}, T_{\text{off}}) \quad T_{\text{on}} = 28.235 \cdot \mu\text{s} \quad T_{\text{off}} = 11.765 \cdot \mu\text{s}$$

Find the range of Inductors that will operate in discontinuous mode

$$D := \frac{T_{\text{on}}}{T_S}$$

$$L := \frac{V_D \cdot T_{\text{on}}}{2 \cdot I_o} \cdot (1 - D)$$

For Discontinuous Mode, We need L less than  $L = 30.727 \cdot \mu\text{H}$

Find the inductor Value

$$L := \frac{V_D^2 \cdot T_{\text{on}}^2}{2 \cdot |V_o| \cdot I_o \cdot T_S} \quad L = 16.609 \cdot \mu\text{H}$$

Find the peak current

$$I_2 := V_D \cdot \frac{T_{\text{on}}}{L} \quad I_2 = 8.5 \cdot \text{amp}$$

Now choose the capacitor

$$\text{Energy dissipated per cycle} \quad E_o := |V_o| \cdot I_o \cdot T_S \quad E_o = 6 \cdot 10^{-4} \cdot \text{joule}$$

$$\text{Specify the ripple desired.} \quad V_{\text{CR}} := 20 \cdot \text{mV}$$

$$\text{Initial Guess} \quad C := 1000 \cdot \mu\text{F}$$

Given

$$\frac{1}{2} \cdot C \cdot (|V_o| + V_{\text{CR}})^2 - \frac{1}{2} \cdot C \cdot (|V_o|)^2 = E_o$$

$$C := \text{find}(C)$$

$$C = 2.498 \cdot \text{mF}$$

Choose the next standard value  $C := 3300 \cdot \mu\text{F}$

Calculate the ripple with the chosen capacitor.

Given

$$\frac{1}{2} \cdot C \cdot (|V_o| + V_{CR})^2 - \frac{1}{2} \cdot C \cdot (|V_o|)^2 = E_o$$

$$V_{CR} := \text{find}(V_{CR}) \quad V_{CR} = 15.142 \cdot \text{mV}$$

### Choose the filter capacitor using the capacitor ESR.

Assume that the major component of the ripple comes from the capacitor ESR

Specify the ripple due to the ESR  $V_{CR} := 20 \cdot \text{mV}$

$$\text{ESR} := \frac{V_{CR}}{I_2} \quad \text{ESR} = 2.353 \cdot 10^{-3} \cdot \Omega$$

For all electrolytic caps, assume that  $\text{ESR} \cdot C = 80 \mu\text{s}$

$$C := \frac{80 \cdot \mu\text{s}}{\text{ESR}} \quad C = 33999.99969 \cdot \mu\text{F}$$

Choose the next size std capacitor  $C := 47000 \cdot \mu\text{F}$

### Summary

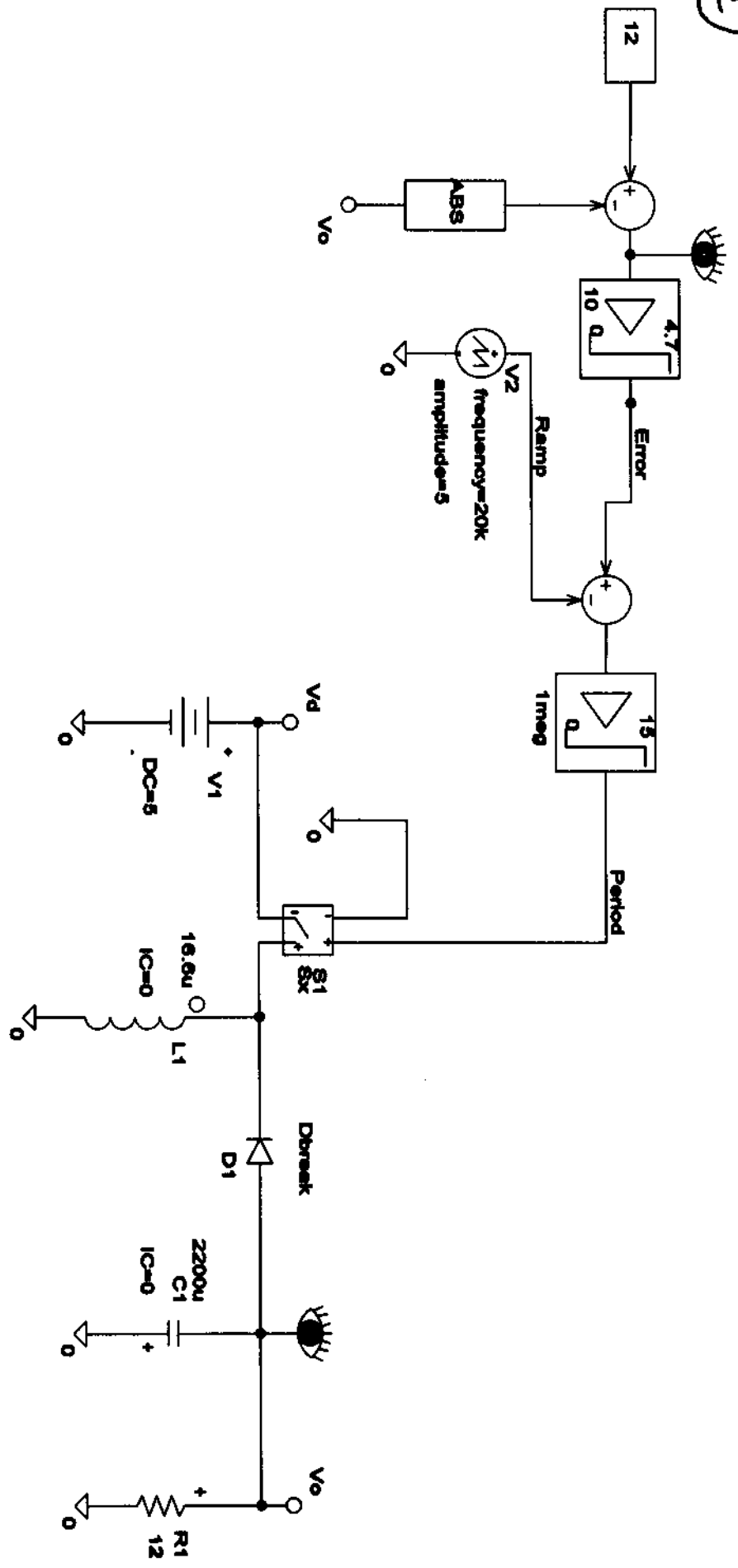
$$L = 16.609 \cdot \mu\text{H} \quad I_2 = 8.5 \cdot \text{amp}$$

$$T_{\text{on}} = 28.235 \cdot \mu\text{s} \quad T_{\text{off}} = 11.765 \cdot \mu\text{s}$$

$$V_D = 5 \cdot \text{volt} \quad V_o = -12 \cdot \text{volt} \quad I_o = 1 \cdot \text{amp}$$

$$C = 4.7 \cdot 10^4 \cdot \mu\text{F} \quad V_{CR} = 20 \cdot \text{mV}$$

142D

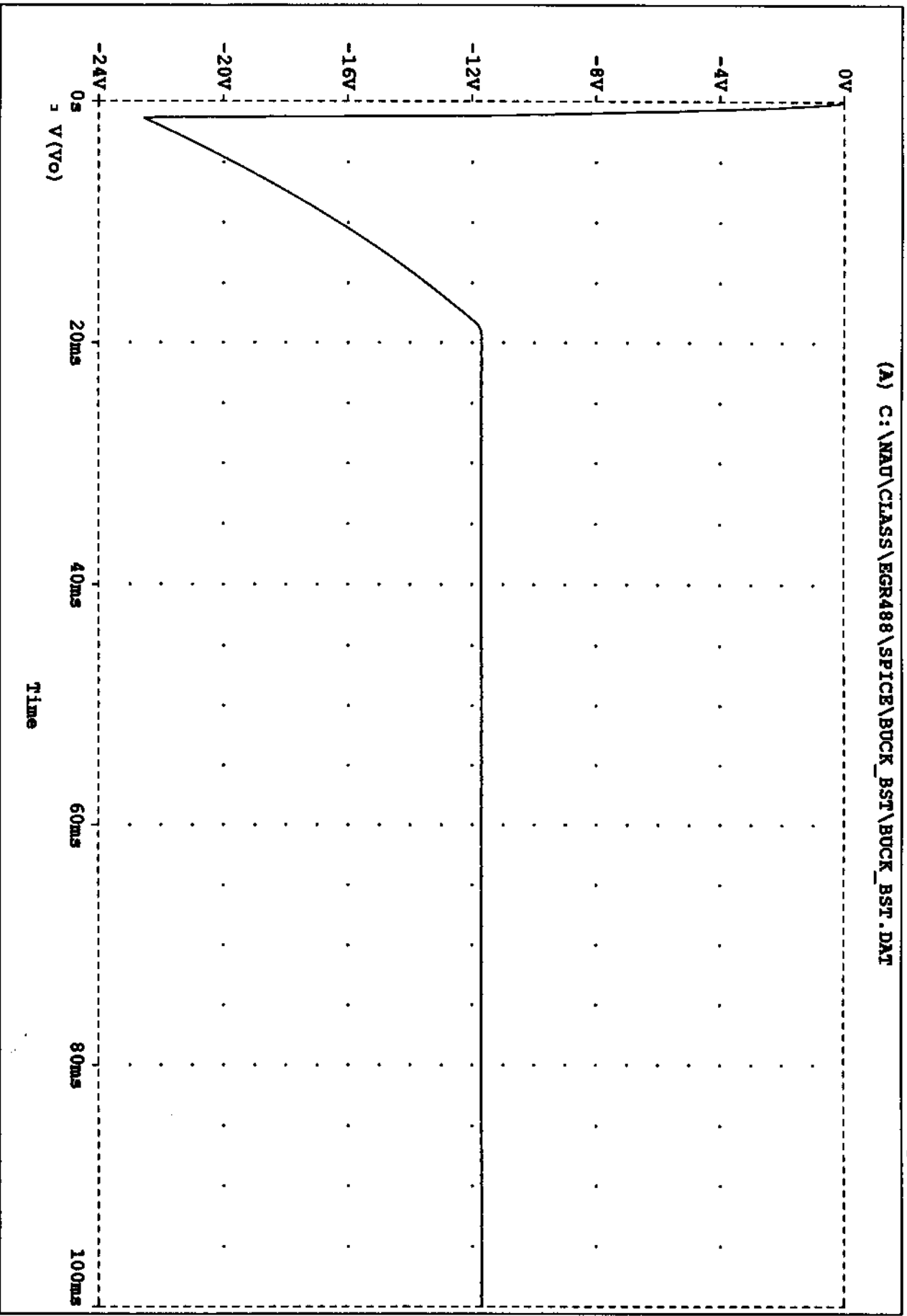


142E

Date/Time run: 10/22/95 15:56:53

Temperature: 27.0

\* C:\NAU\CLASS\EGR488\SPICE\BUCK\_BST\BUCK\_BST.SCH  
(A) C:\NAU\CLASS\EGR488\SPICE\BUCK\_BST\BUCK\_BST.DAT



Date: October 22, 1995

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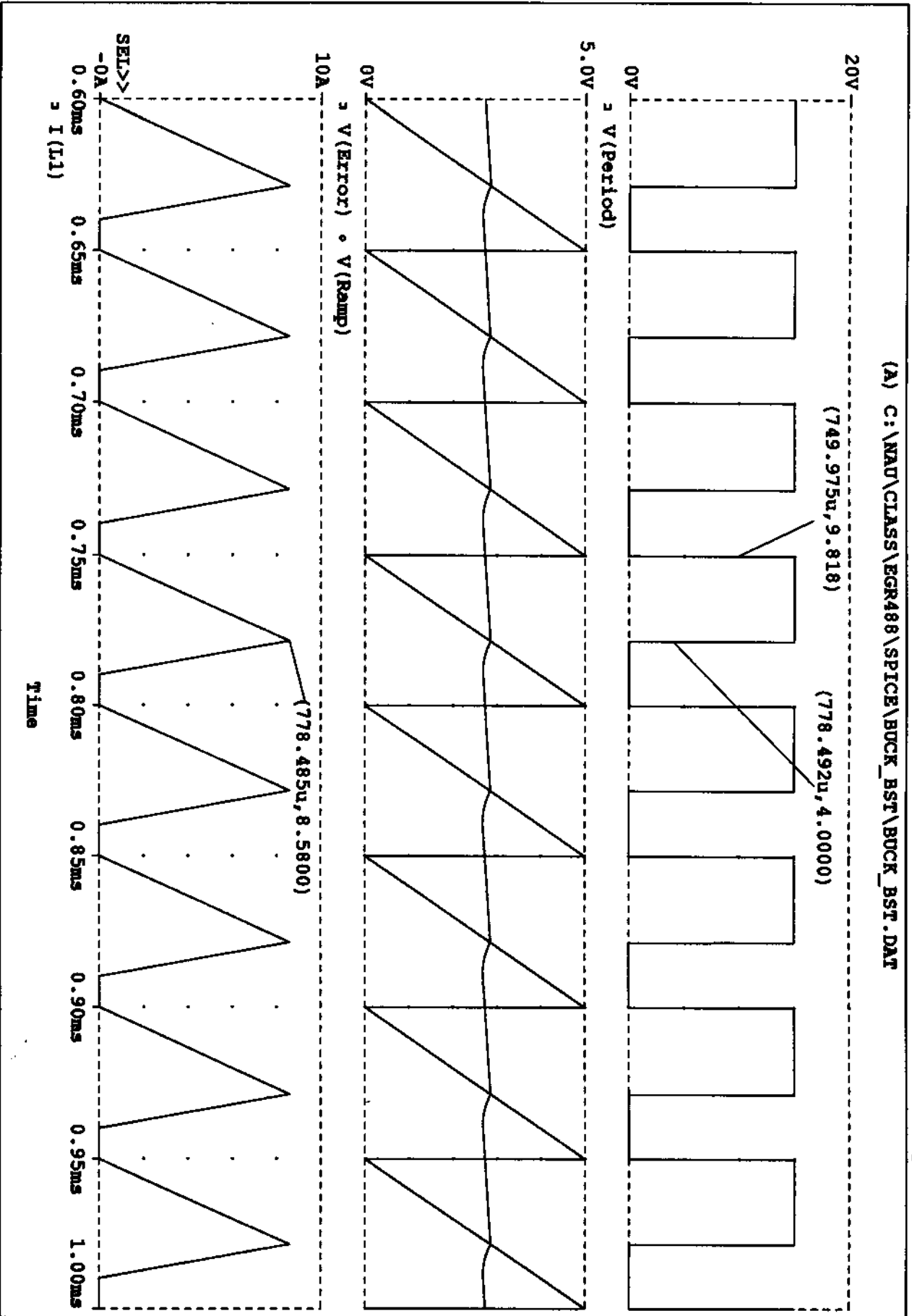
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142F

Date/Time run: 10/22/95 14:38:43

Temperature: 27.0

\* C:\NAU\CLASS\EGR488\SPICE\BUCK\_BST\BUCK\_BST.SCH  
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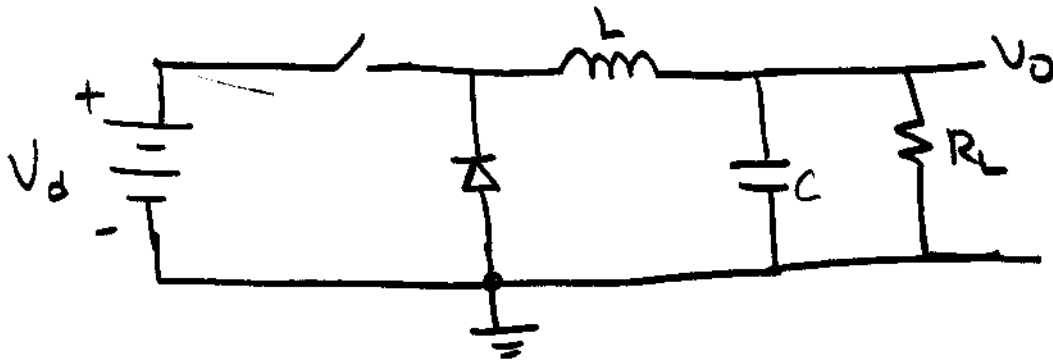


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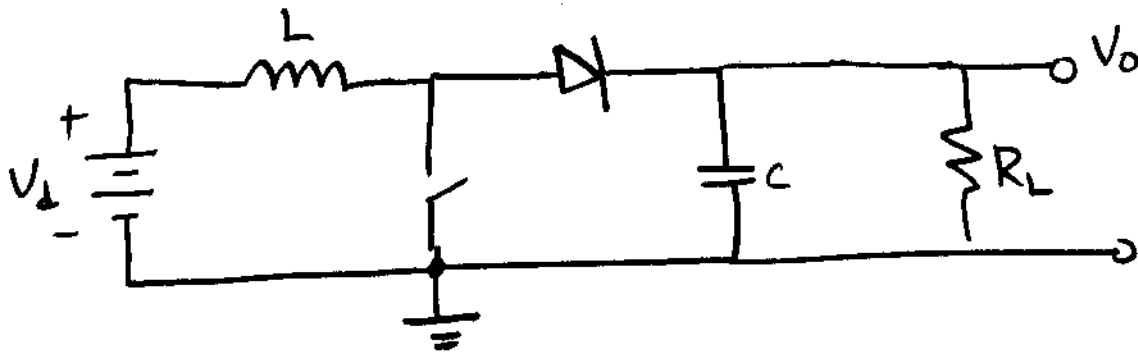
Page 1

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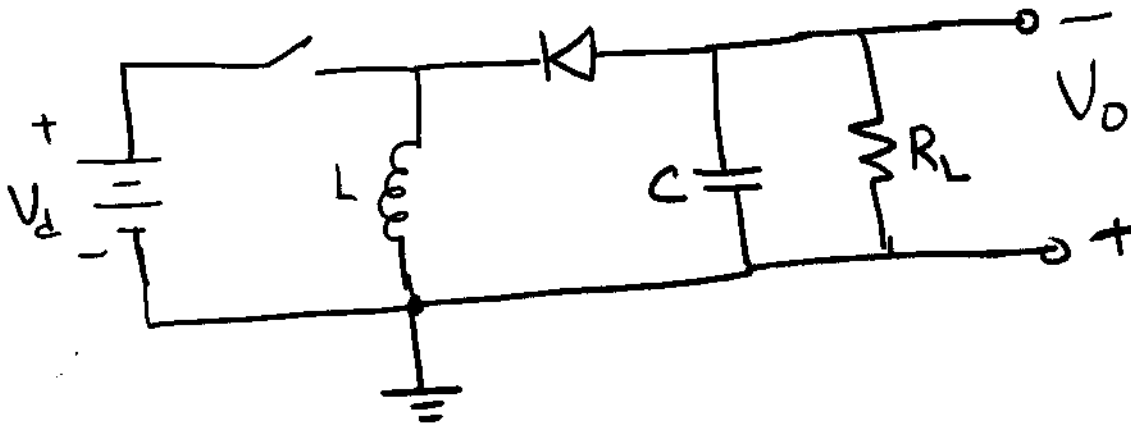
# SUMMARY



Buck



Boost



Buck-Boost

Buck

$$V_0 = DV_0$$

$$I_0 = I_0 \left( \frac{1}{D} \right)$$

$$I_1 + I_2 = 2I_0$$

$$I_2 - I_1 = \left( \frac{V_L - V_0}{L} \right) t_{on}$$

$$I_0 \geq \left( \frac{V_0 - V_0}{2L} \right) t_{on}$$

CONTINUOUS MODE

BOOST

$$V_0 = V_d \left[ \frac{1}{1-D} \right]$$

$$I_0 = I_0 [1-D]$$

$$I_1 + I_2 = \frac{2I_0}{1-D}$$

$$I_2 - I_1 = \frac{V_d t_{on}}{L}$$

$$I_0 \geq \frac{V_d t_{on}}{2L} (1-D)$$

Buck-Boost

$$V_0 = V_d \left[ \frac{D}{1-D} \right]$$

$$I_0 = I_0 \left[ \frac{1-D}{D} \right]$$

$$\frac{I_1 + I_2}{D} = \frac{2I_0}{1-D}$$

$$I_2 - I_1 = \frac{V_d t_{on}}{L}$$

$$I_0 \geq \frac{V_d t_{on}}{2L} (1-D)$$

DISCONTINUOUS MODE

BUCK

$$I_2 = \frac{(V_d - V_o)}{L} t_{on}$$

$$I_2 = \frac{V_o t_{off}}{L}$$

$$\frac{t_{on}}{t_{off}} = \frac{V_o}{V_d - V_o}$$

$$V_o = \frac{t_{on}^2 V_o^2}{2 I_o T_s L + V_o t_{on}^2}$$

$$I_o \leq \left( \frac{V_o - V_o}{2L} \right) t_{on}$$

BOOST

$$I_2 = \frac{V_d t_{on}}{L}$$

$$I_2 = \frac{(V_o - V_d)}{L} t_{off}$$

$$\frac{t_{on}}{t_{off}} = \frac{V_o - V_d}{V_d}$$

$$t_{on} + t_{off} \leq 0.8 T_s$$

$$V_o = \frac{V_d^2 t_{on}}{2L T_s I_o} (t_{on} + t_{off})$$

$$I_o \leq \left( \frac{V_d t_{on}}{2L} \right) \left( \frac{V_d}{V_o} \right)$$

BUCK-BOOST

$$I_2 = \frac{V_d t_{on}}{L}$$

$$I_2 = \frac{V_o t_{off}}{L}$$

$$\frac{t_{on}}{t_{off}} = \frac{V_o}{V_d}$$

$$t_{on} + t_{off} \leq 0.8 T_s$$

$$V_o = \frac{V_d^2 t_{on}^2}{2L I_o T_s}$$

$$I_o \leq \frac{V_d t_{on}}{2L} (1-D)$$